

# Breaching 1.5°C: Give me the odds

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2024-12-17

## Abstract

Climate change communication is crucial to raising awareness and motivating action. In the context of breaching the limits set out by the Paris Agreement, we argue that climate scientists should move away from point estimates and towards reporting probabilities. Reporting probabilities will provide policymakers with a range of possible outcomes and will allow them to make informed timely decisions. To achieve this goal, we propose a method to calculate the probability of breaching the limits set out by the Paris Agreement. The method can be summarized as predicting future temperatures under different scenarios and calculating the number of possible outcomes that breach the limits as a proportion of the total number of outcomes. The probabilities can be computed for different time horizons and can be updated as new data become available. As an illustration, we performed a simulation study to investigate the probability of breaching the limits in a statistical model. Our results show that the probability of breaching the 1.5°C limit is already greater than zero for 2024. Moreover, the probability of breaching the limit is greater than 99% by 2042 if no action is taken to reduce greenhouse gas emissions. Our methodology is simple to implement and can easily be extended to more complex models of the climate system. We encourage climate model developers to include the probabilities of breaching the limits in their reports.

## The 1.5°C limit

The goals of the Paris Agreement (PA) have recently gained renewed media attention due to observed temperature anomalies that exceeded 1.5°C above preindustrial levels for 12 consecutive months according to Copernicus Climate Change Service (2024a). The importance of the 1.5°C threshold is that it was established in the PA as a limit to avoid the most severe consequences of climate change. Formally, the PA aims to limit global warming to well below 2°C above pre-industrial levels and to pursue efforts to limit the temperature increase to 1.5°C.

An obstacle in assessing the success or failure of the PA is the lack of a clear definition of when temperature limits are breached (Betts et al. 2023). The definition of when the limits are breached is crucial for both scientific and political reasons.

**If we defined the breaching of 1.5°C as the mean temperature for a year being**

35 **above that limit, it has already been breached.**

36 However, to avoid short-term fluctuations, the Sixth Assessment Report of Working Group I  
37 of the Intergovernmental Panel on Climate Change (IPCC) proposes to use a 20-year average  
38 temperature rise to determine when the limit is exceeded (IPCC 2021). The question remained  
39 on when inside that 20-year period the limit is breached.

40 Betts et al. (2023) argue that defining the breach of the 1.5°C limit as the last year in a 20-year  
41 period where the global mean temperature is above that limit delays the conclusion of a breach  
42 by a decade. They propose using the midpoint of the 20-year period as the year when the limit  
43 is breached. Thus, computing when the threshold will be breached entails averaging several  
44 years of observed temperature rise with a forecast of the following years up to the 20-year  
45 period. We extend this methodology to provide the probability of breaching the 1.5°C and 2°C  
46 limits with the aim of improving the communication of climate change.

#### 47 **Improving communication of climate change**

48 One of the main challenges in communicating climate change is the complexity of the topic. This  
49 complexity makes it difficult to communicate the issue in a way that is easily understandable to  
50 the general public. In the context of breaching the limits set out by the PA, communication is  
51 crucial. The issue can become highly politicized if not communicated effectively. The public and  
52 policymakers need timely information about the urgency of the situation and the consequences  
53 of inaction.

54 One of the first steps in improving communication is to provide data in a clear and under-  
55 standable way. Datasets report temperature anomalies as the difference between the observed  
56 temperature and the average temperature for a reference period (GISTEMP 2020; Morice et  
57 al. 2021; R. A. Rohde and Hausfather 2020). Even though the PA states that the reference  
58 period should be pre-industrial levels, the datasets typically use a more recent reference period.  
59 For example, the HadCRUT5 dataset uses the 1961-1990 average temperature as the reference  
60 period.

## Temperature anomalies

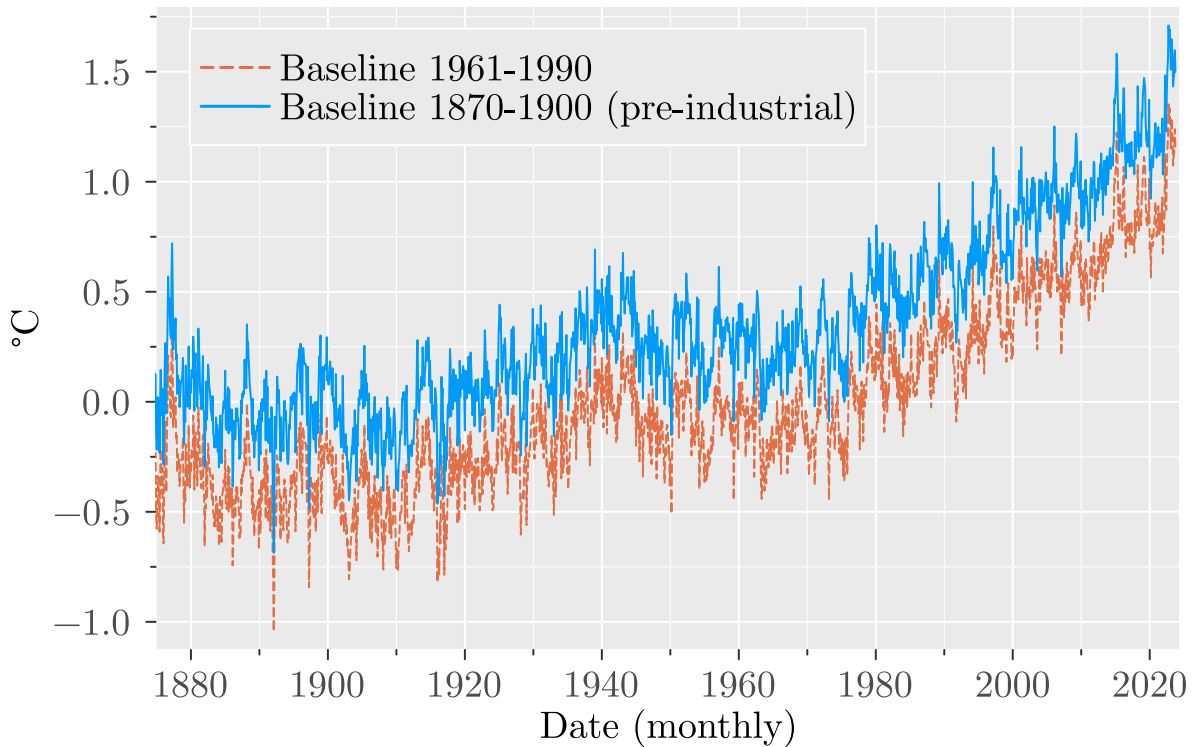


Figure 1: Temperature anomalies ( $^{\circ}\text{C}$ ) in the HadCRUT5 dataset (Morice et al. 2021). The dashed line presents the data according to the baseline period in HadCRUT5 (1961-1990). The solid line represents the temperature anomalies with the pre-industrial baseline period (1870-1900).

<sup>61</sup> Source: [Breaching 1.5°C: Give me the odds](#)

<sup>62</sup> Figure 1 shows temperature anomalies as reported by the HadCRUT5 dataset. The figure  
<sup>63</sup> shows that if we use the 1961-1990 average temperature as the reference period, as presented in  
<sup>64</sup> the dataset, the temperature anomalies have not breached the 1.5°C limit yet. However, if we  
<sup>65</sup> use the pre-industrial levels as the reference period, as indicated in the PA, the limit has already  
<sup>66</sup> been breached several times. This mismatch between the reference period used in the datasets  
<sup>67</sup> and the reference period in the PA can lead to misunderstandings and misinterpretations. A  
<sup>68</sup> sceptic reading a news article reporting temperature anomalies breaching the 1.5°C limit above  
<sup>69</sup> pre-industrial levels can easily download and plot the data getting the impression that the  
<sup>70</sup> headline is an exaggeration if they are not aware of the reference period used.

<sup>71</sup> **All datasets should use the same reference period based on the pre-industrial levels.**  
<sup>72</sup> This will help to avoid confusion and to make it easier to compare the data. However, for  
<sup>73</sup> historical reasons, data providers should also report temperature anomalies relative to their

74 original reference period. This will help maintain compatibility with previous reports and  
75 models trained on the original data.

## 76 **Predictions for the breaching of the PA limits**

77 It should be stressed in any report that determining when the 1.5°C limit will be breached  
78 requires forecasting future temperatures. Forecasts can take many forms. The most common  
79 are physical models that simulate the climate system [see e.g.; Nath et al. (2022); Eyring  
80 et al. (2016); Held et al. (2019); Collins, Tett, and Cooper (2001); Orbe et al. (2020)].  
81 Physics-based models are computationally expensive and require high-performance computing.  
82 Hence, reduced-complexity models have been developed. These models are based on statistical  
83 methods and are trained on historical data of different climate variables [see e.g.; Meinshausen,  
84 Raper, and Wigley (2011); Smith et al. (2024); Bennedsen, Hillebrand, and Koopman (2024)].  
85 Regardless of the method used to predict future temperatures, forecasts are uncertain. The  
86 climate system is complex and chaotic. This complexity is reflected in the confidence intervals  
87 associated with the forecasts. For example, the IPCC provides a range of possible outcomes for  
88 future temperatures. However, the uncertainty in the forecasts is not communicated effectively  
89 when discussing breaching the limits set out by the PA.

90 The media has recently reported new estimates on when the 1.5°C limit will be breached  
91 (Copernicus Climate Change Service 2024b; R. Rohde 2024). However, these estimates are  
92 often presented as point estimates without confidence intervals or without a clear description  
93 of the methodology used to make the predictions. In the current political environment, it is  
94 crucial to communicate the uncertainty in the predictions.

95 Recent point estimates of when the 1.5°C limit will be breached can be counterproductive if  
96 not accompanied by probability estimates. In case the limit is not breached in precisely the  
97 year predicted, it can give climate change deniers an argument to dismiss scientific evidence.  
98 In the past, extreme winters have been used as an argument against global warming due to  
99 the misunderstanding of the difference between weather and climate. Where weather refers to  
100 something more local and only observed over short-time periods, climate is more long-termed.  
101 **The distinction between weather and climate must be clear in any communication**  
102 **to avoid misrepresentation of the results.**

## 103 **A new methodology to measure when we will breach the limit of 1.5°C**

104 We propose a way to communicate the uncertainty in the predictions of when the limits set at  
105 the PA will be breached. The methodology builds on the proposal by Betts et al. (2023) to  
106 use a 20-year average temperature rise centered around a particular year. The 20-year average  
107 is then compared with the 1.5°C and 2°C limits. We use models to produce multiple scenarios  
108 of future temperature rise and compute the number of scenarios that breach the limits as a  
109 proportion of the total number of scenarios. The probabilities can be computed for different  
110 time horizons and can be updated as new data become available. Moreover, the methodology  
111 can be easily applied for different climate models and datasets.

112 There are already several examples of how probabilities can be used to communicate climate  
113 change effectively [see e.g.; IPCC (2021); Wigley and Raper (2001); S. H. Schneider (2001); S. H.  
114 Schneider and Mastrandrea (2005); T. Schneider et al. (2023)]. **By reporting probabilities,**  
115 **we can communicate the uncertainty in the predictions and provide policymakers**  
116 **with a range of possible outcomes.** This will allow policymakers to make more informed  
117 decisions on taking action to reduce greenhouse gas emissions. Reporting in 2024 a probability  
118 of 50% that the limit will be breached in 2030 will give an indication of the urgency of the  
119 situation. The probability distribution will also reflect how the odds of avoiding the breach  
120 decrease over time if no action is taken. This will provide a clear picture of the consequences  
121 of delaying action.

122 To illustrate our methodology, we developed a simulation study. We simulate multiple scenarios  
123 of future temperature rise and calculate the probability of breaching the 1.5°C and 2°C limits.  
124 The simulation study is presented next.

## 125 **A statistical model to predict future temperatures**

126 **Data.** The data used in this paper is the global mean temperature anomaly of the HadCRUT5  
127 dataset computed by the Met Office Hadley Centre Morice et al. (2021). The data are reported  
128 as the difference between the observed temperature and the 1961-1990 average temperature and  
129 are available from 1850. We first convert the data to anomalies compared to pre-industrial levels.  
130 The pre-industrial levels are defined as the average temperature from the earliest available data  
131 up to 1900. The data is presented in Figure 1.

132 HadCRUT5 provides 200 realizations to account for the uncertainty in the data. We use all

133 realizations to fit the models and produce multiple scenarios of future temperature rise. This  
134 allows us to account for the uncertainty in the data and to provide a range of possible outcomes.  
135 We fit the models to each realization separately and produce five different scenarios of future  
136 temperatures for each realization. This gives us a total of 1000 scenarios of future temperatures.  
137 The methodology can be easily extended to include more realizations and scenarios.

138 **Modeling scheme.** Our modeling scheme consists of three components: a trend specification,  
139 an El Niño Southern Oscillation (ENSO) model, and a long-range dependent error term. We  
140 provide a brief overview of the models. Further technical details on the models are presented  
141 in the supplementary material in the appendix, and the code used to perform the simulation  
142 study is available in a [Jupyter notebook in the supplementary material](#).

143 We consider three trend specifications for modeling the global mean temperature anomaly: a  
144 linear trend model, a quadratic trend model, and a linear trend that allows for a break. The  
145 models are estimated on the historical temperature data. The best model is selected on the  
146 basis of the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC)  
147 (Akaike 1974; Schwarz 1978). For each realization, the model with the lowest AIC and BIC is  
148 considered the best model and is used to predict future temperatures.

149 Furthermore, we control for the El Niño effect as it is known to have an effect on the global  
150 mean temperature anomaly (Thirumalai et al. 2017; Jiang et al. 2024). To control for the El  
151 Niño effect we include the Oceanic Niño Index (ONI) as a covariate in the models. The ONI is  
152 an indicator for monitoring the ENSO. El Niño conditions are present when the ONI is +0.5  
153 or higher. Oceanic La Niña conditions exist when the ONI is -0.5 or lower.

154 For forecasting purposes, we fit a Markov-switching model to the ONI data to predict future  
155 values (Hamilton 1989, 1990). The motivation for using a Markov-switching model is that the  
156 ONI data naturally exhibit regime changes over time. The number of states in the Markov-  
157 switching model is 7, which is selected on the basis of the AIC and BIC. The seven states  
158 correspond to the different phases of the ENSO cycle, ranging from very strong El Niño, strong  
159 El Niño, moderate El Niño, neutral, moderate La Niña, strong La Niña, to very strong La Niña.  
160 Finally, our modeling scheme allows for the error term to have long-range dependence. Long-  
161 range dependence has its origin in the analysis of climate data (Hurst 1956). Temperature data  
162 are known to have long-range dependence, which means that the error terms are correlated  
163 over long periods (Bloomfield and Nychka 1992; Bloomfield 1992; J. Eduardo Vera-Valdés

164 2021). The long-range dependence parameter is estimated using the exact local Whittle method  
165 (Shimotsu and Phillips 2005).

166 **Model validation.** We obtain the prediction intervals for temperature anomalies using our  
167 modeling scheme fitted to data up to November 2016, the month when the PA entered into  
168 force. All HadCRUT5 realizations are considered. The results, presented in the supplement-  
169 ary material, show that our models provide adequate coverage of the observed temperature  
170 anomalies up to the present day. We take this as validation of our modeling scheme.

171 **Model fitting.** As an illustration, we present a fitted model and its forecast for realization  
172 10 of the HadCRUT5 dataset. Realization 10 is chosen arbitrarily. The model is fitted to the  
173 data up to the last observation. The model is then used to forecast future temperatures. The  
174 results are presented in Figure 2.

### Forecast for realization 10 of HadCRUT5

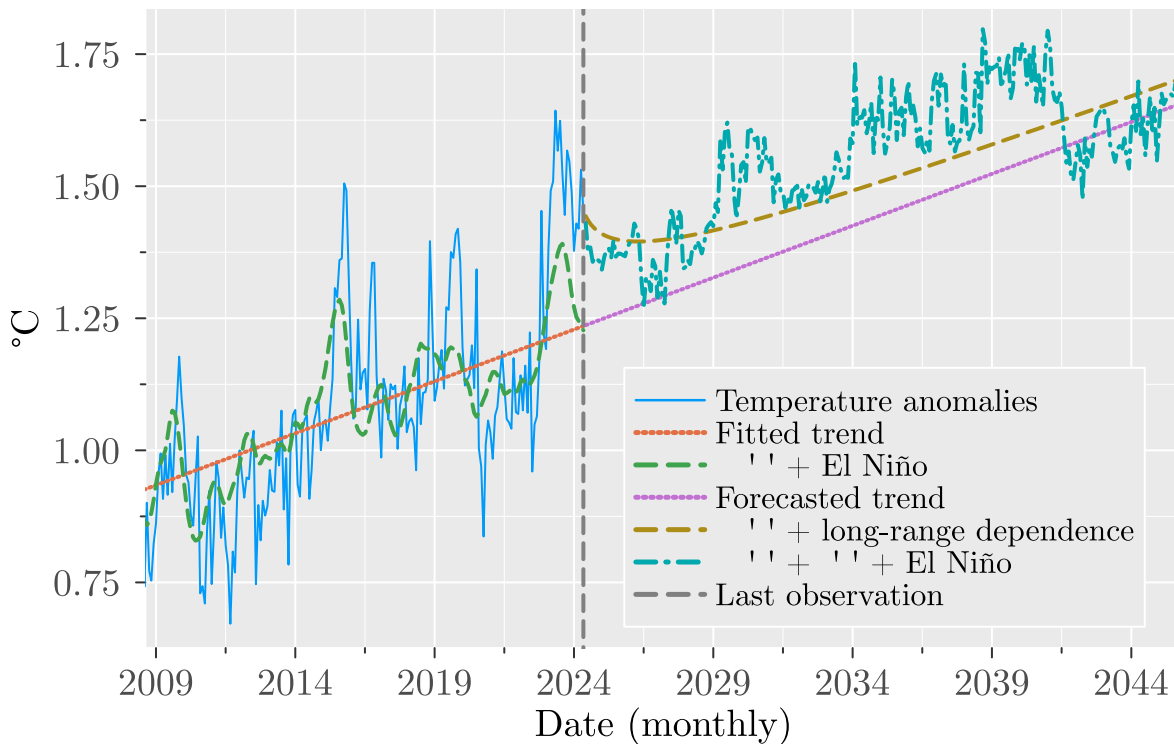


Figure 2: Forecast of temperature anomalies for realization 10 of the HadCRUT5 dataset. The forecast is based on the broken trend model with long-range dependence and El Niño as an exogenous variable. A simulated El Niño series using a Markov-switching model with 7 states was used to generate the forecast.

175 Source: [Breaching 1.5°C: Give me the odds](#)

176 Figure 2 highlights the different components of the model: the trend, the long-range dependence,  
177 and the El Niño effect.

178 The trend component captures the long-term increase in the temperature anomaly, all other  
179 things being equal. The long-range dependence captures the persistence of the temperature  
180 anomaly over time. Given that recent temperatures are high, the long-range dependence in  
181 the data implies that future temperatures are likely to remain high. This directly affects the  
182 forecasted temperature and the probability of breaching the limits. Finally, the El Niño effect  
183 captures the short-term fluctuations in the temperature anomaly. The forecasted temperature  
184 anomaly is the sum of the trend, the long-range dependence, and the El Niño effect.

185 **Breaching the limits.** For each simulated path, we calculate the average temperature for  
186 20 years using a moving average. We began the process in 2004 to obtain a 20-year average  
187 temperature rise centered around 2014 and with an end point in the current year. The moving  
188 average is then calculated for each month. We repeat this process until the end of the forecasted  
189 period. We then find the first month where the 20-year average temperature rise breaches the  
190 1.5°C and 2°C limits.



## Breaching of the 1.5°C threshold for realization 10

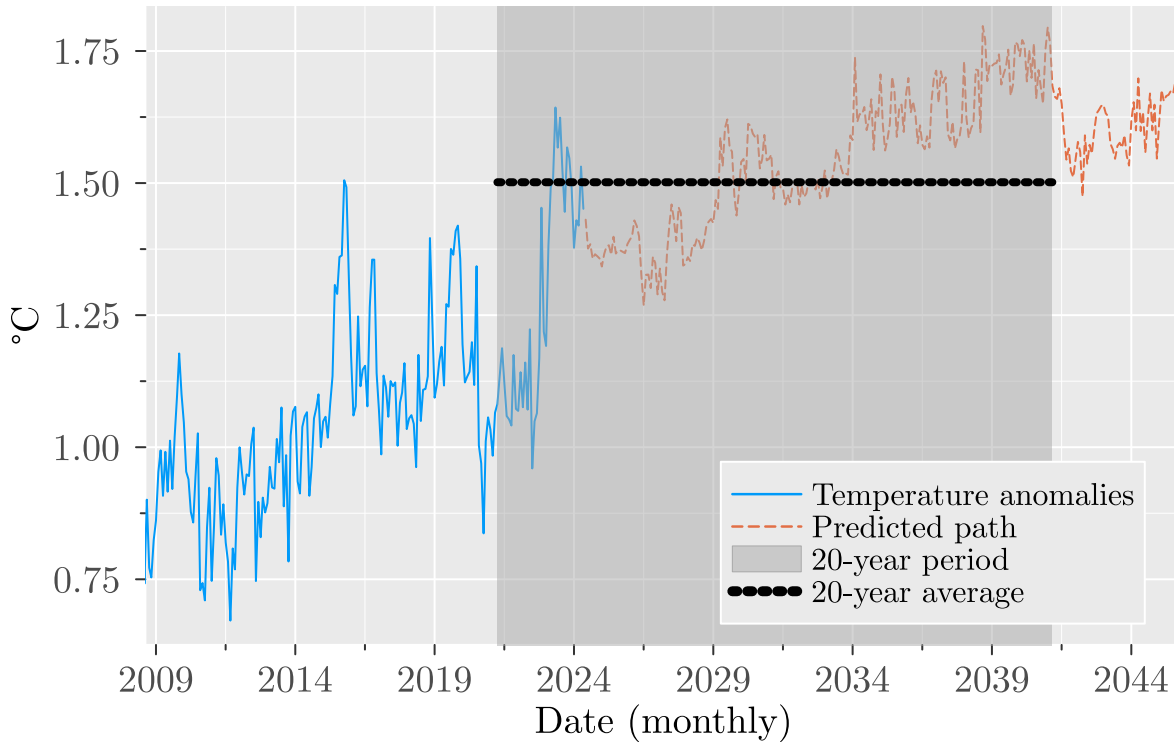


Figure 3: Breaching of the 1.5°C threshold for realization 10 of the HadCRUT5 dataset. The figure shows the temperature anomalies and the forecasted path for the next several months. The 20-year period is highlighted in gray, and the 20-year average is shown as a black dashed line.

191 Source: [Breaching 1.5°C: Give me the odds](#)

192 Figure 3 shows that the 20-year average temperature for the simulated path of realization 10  
193 first breaches the 1.5°C limit in July of 2031. The gray box indicates the 20-year period used  
194 to calculate the average temperature rise, while the black dashed line indicates the 20-year  
195 average temperature.

196 The month in which the limit is breached for this path is highly dependent on the El Niño effect.  
197 Hence, we conduct a simulation study to estimate the probability of breaching the limits.

### 198 **Simulation study**

199 Using the modeling scheme described above, we detail a way to compute the probability of  
200 breaching the limits set out by the PA using a simulation study. The use of Monte Carlo meth-  
201 ods, as the one used in this simulation study, is a common approach to estimate probabilities  
202 in complex systems, and it is pursued by the IPCC (Abel, Eggleston, and Pullus 2002). The

203 simulation study has two main steps.

204 First, we forecast the global mean temperature anomaly using the best model selected using  
205 the information criteria. For each realization of the HadCRUT5 dataset, we simulate 5 different  
206 scenarios of future temperature rise by simulating different paths for El Niño effect. This gives  
207 us a total of 1000 scenarios of future temperatures. Figure 4 shows the simulated temperature  
208 anomalies for a subset of the realizations to simplify visualization and plot rendering.

### Simulated forecast paths for temperature anomalies

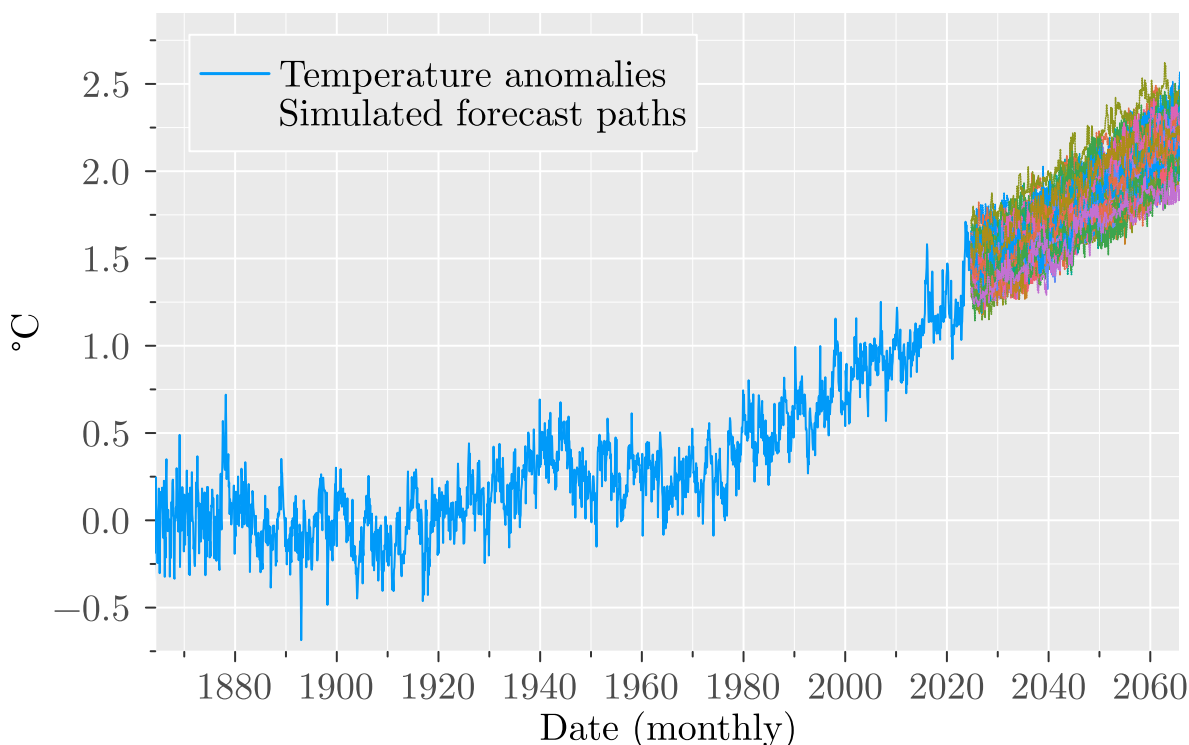


Figure 4: Simulated forecast paths for HadCRUT5 temperature anomalies. One hundred paths of a total of 1000 paths are shown to ease visualization. The forecasts are based on the best-fitting model for each realization, with El Niño as an exogenous variable. For ease of visualization, the mean of all temperature anomaly realizations is shown as a solid line.

209 Source: [Breaching 1.5°C: Give me the odds](#)

210 In a second step, we calculate the 20-year moving average centered around a particular month  
211 for each simulated path. We repeat this process for all simulated paths and recover the ratio  
212 of paths that breach the 1.5°C and 2°C limits each month to the total number of paths. We  
213 then plot this proportion of paths that crossed either threshold to obtain an estimate of the  
214 probability of breaching the limits. Figure 5 presents the results of the simulation study.

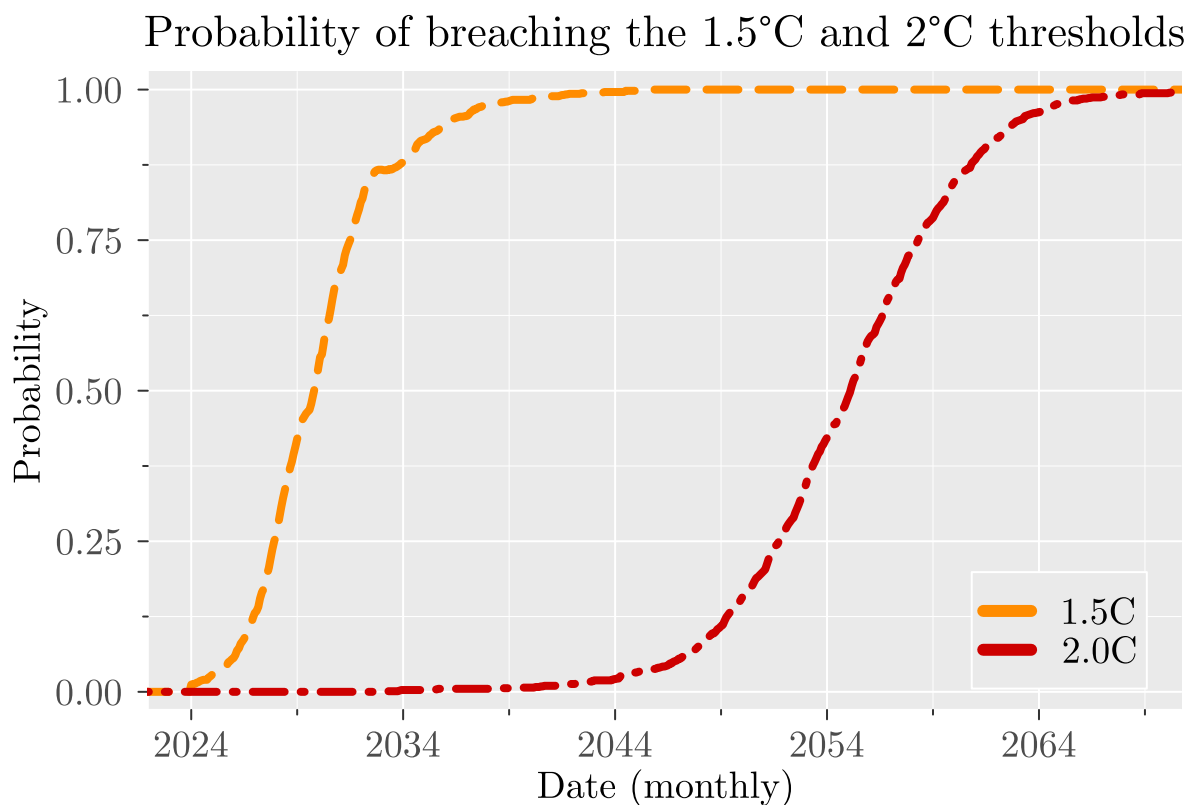


Figure 5: Proportion of scenarios that breach the 1.5°C and 2°C thresholds for the HadCRUT5 temperature anomalies for each month. The figure considers 1000 scenarios, each based on the best-fitting model for each realization, with five simulations for El Niño as an exogenous variable each.

<sup>215</sup> Source: [Breaching 1.5°C: Give me the odds](#)

<sup>216</sup> Some key results from the simulation study are presented in Table 1. The table shows the first  
<sup>217</sup> month the 1.5°C and 2°C limits are breached at a given probability level. The results are based  
<sup>218</sup> considering the 20-year average temperature.

Table 1: Months to breach the 1.5°C and 2°C thresholds for the HadCRUT5 temperature anomalies at a given probability level.

Probability level and period	1.5°C threshold	2°C threshold
Above 0%, 20-years avg.	2024-09-01	2033-11-01
Above 50%, 20-years avg.	2030-07-01	2055-11-01
Above 99%, 20-years avg.	2042-02-01	2068-04-01
Above 0%, 30-years avg.	2029-09-01	2040-04-01
Above 50%, 30-years avg.	2035-08-01	2060-11-01

Probability level and period	1.5°C threshold	2°C threshold
Above 99%, 30-years avg.	2046-12-01	2072-12-01

219 **The simulation study considered here shows that the probability of breaching the**  
 220 **1.5°C limit is already greater than zero for 2024.**

221 This means that there is at least one scenario in which the 20-year average temperature rise  
 222 breaches the 1.5°C limit in September 2024. Moreover, note that there is a rapid increase in  
 223 the probability of breaching the 1.5°C limit after 2030. The probability of breaching the limit is  
 224 already greater than 50% by July 2030. This is in line with recent predictions that the goal will  
 225 likely be breached in the second half of the 2030 decade (Copernicus Climate Change Service  
 226 2024b; R. Rohde 2024). Our simulation study provides an estimate of the monthly probabilities  
 227 of breaching the goals. They show that the probability of breaching the 1.5°C limit is greater  
 228 than 99% by 2042 if no action is taken to reduce greenhouse gas emissions.

229 Regarding the 2°C limit, our simulation study finds that the probability of breach already  
 230 starts increasing above zero by the 2030 decade. In general, the simulation study highlights  
 231 that climate change mitigation policies should be implemented as soon as possible to avoid  
 232 breaching the limits set by the PA.

233 Furthermore, Table 1 shows the breaching probabilities considering a 30-year average. The  
 234 motivation for considering the 30-year average temperature is that baseline periods for climate  
 235 data are often defined as 30-year averages (Morice et al. 2021; GISTEMP 2020; R. A. Rohde and  
 236 Hausfather 2020). Moreover, some studies use the 30-year average temperature to determine  
 237 when the limits are breached (Copernicus Climate Change Service 2024b). The results are

### 238 **How has the probabilities changed since the Paris Agreement**

239 As model validation, Figure 6 presents the prediction intervals for temperature anomalies for  
 240 the modeling scheme described above starting in November 2016, the month when the PA  
 241 entered into force. The results using the data up to the PA are presented in the [supplementary](#)  
 242 [Jupyter notebook](#).

## Simulated forecast paths for temperature anomalies

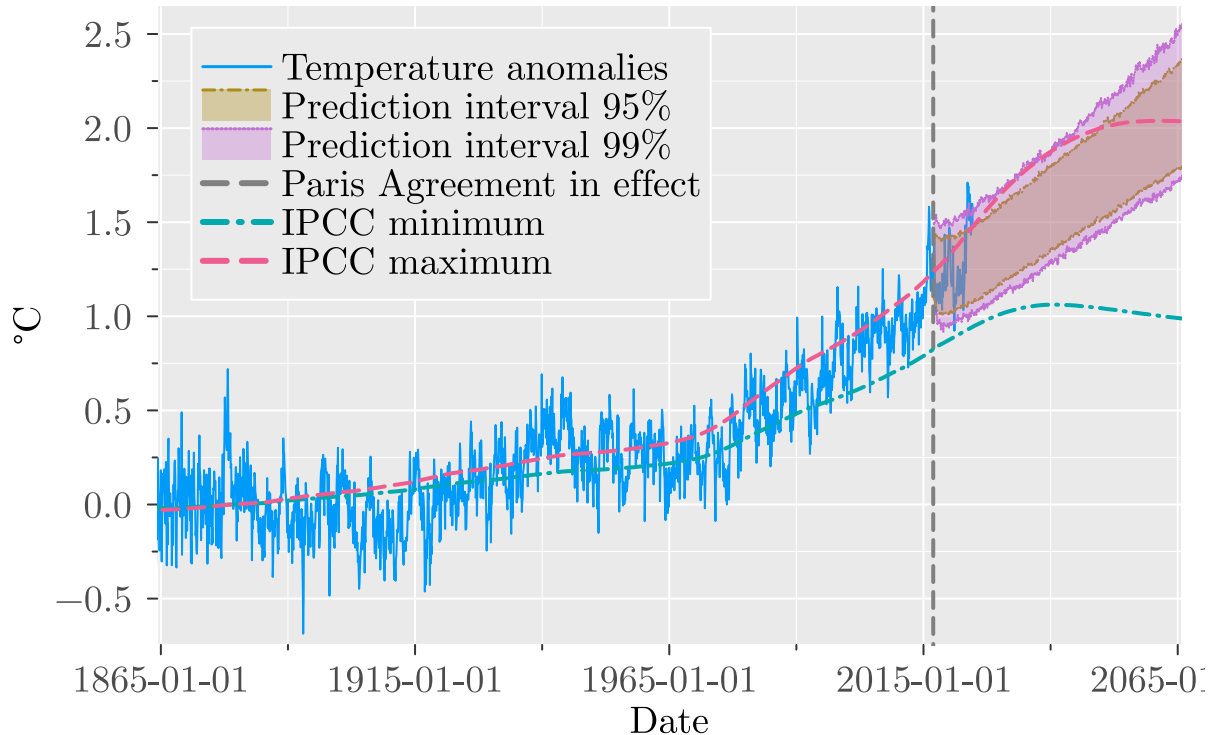


Figure 6: Simulated forecast paths for HadCRUT5 temperature anomalies. The 95% and 99% prediction intervals are shown as shaded areas. The IPCC projections for the minimum and maximum temperature anomalies are shown as dashed lines (Allen et al. 2018).

243 Source: [Breaching 1.5°C: Give me the odds](#)

244 The prediction intervals are based on the historical data up to the start of the PA and the  
245 models fitted to the data. The prediction intervals are used to assess the uncertainty in the  
246 forecasts. In general, the prediction intervals provide adequate coverage of the observed temper-  
247 ature anomalies. However, note that recent high temperatures fall outside the 99% prediction  
248 intervals. This further signals the abnormality of the recent temperature observations. Several  
249 theories have been proposed to explain recent high temperatures, including decreased cloud  
250 coverage and international shipping regulatory changes (Goessling, Rackow, and Jung; Quaglia  
251 and Visioni 2024). Regardless of the cause, the high temperatures highlight the urgency of the  
252 situation.

253 In contrast, the figure presents the temperature projections from the summary for policymakers  
254 of the IPCC Special Report: Global Warming of 1.5°C (Allen et al. 2018). The paths show the  
255 projected temperature evolution according to the IPCC if CO<sub>2</sub> emission gradually decrease to

256 zero by 2055, while other greenhouse gas levels stop changing after 2030. The figure shows that  
 257 recent temperatures are outside the IPCC projections. Hence, the IPCC projections coverage  
 258 is lacking, and the projections are likely to be too optimistic.

259 Furthermore, Figure 7 presents the probabilities of breaching the 1.5°C and 2°C limits at the  
 260 start of the PA.

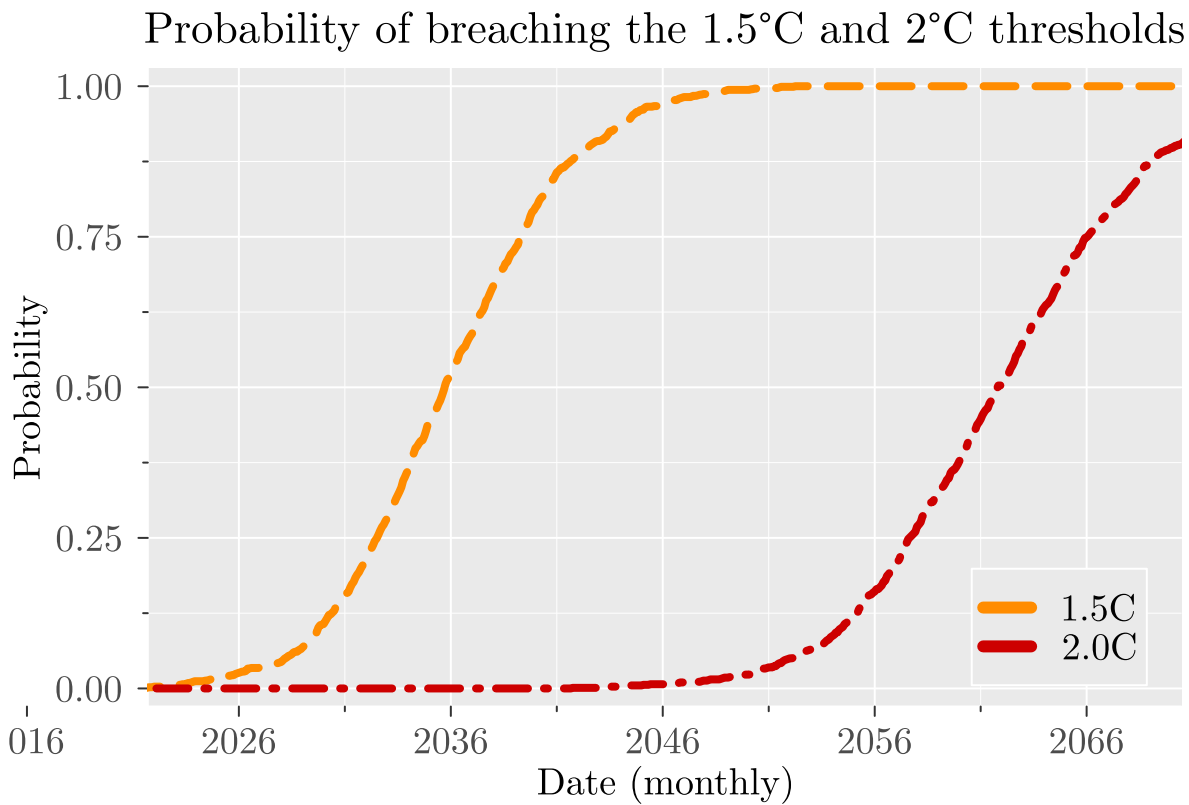


Figure 7: Proportion of scenarios that breach the 1.5°C and 2°C thresholds for the HadCRUT5 temperature anomalies for each month at the start of the Paris Agreement. The figure considers 1000 scenarios, each based on the best-fitting model for each realization, with five simulations for El Niño as an exogenous variable each.

261 Source: [Breaching 1.5°C: Give me the odds](#)

262 The figure allows us to assess how the probability of breaching the limits has changed since the  
 263 PA. At the start of the PA, the probability of breaching the 1.5°C limit with a probability of  
 264 99% was not encountered until 2051. The probability of breaching the 2°C limit at a probability  
 265 of 99% was not encountered in the forecast period ending in 2083. The results are related to the  
 266 exercise of Copernicus Climate Change Service (2023) on the time *lost* since the PA considering  
 267 a point estimate, while we provide the probabilities of breaching the limits. Probabilities have  
 268 increased significantly since the PA, which highlights that the urgency of the situation has

269 increased since the PA.

## 270 **Discussion and further work**

271 We have presented a new way to communicate when we will breach the temperature limits  
272 set out by the PA. Our methodology is simple to implement. It requires predicting future  
273 temperatures under different scenarios and calculating the number of possible outcomes that  
274 breach the limits as a proportion of the total number of outcomes. The probabilities can be  
275 computed for different time horizons and datasets and can be updated as new data becomes  
276 available. Additional simulation exercises considering alternative datasets and sub-samples of  
277 realizations are presented in the supplementary material. They show that the breaching dates  
278 are robust to the choice of dataset. Moreover, and additional analysis of the probabilities of  
279 breaching the limits since the PA is presented in the supplementary material. It shows that  
280 the probabilities have increased significantly since the PA, highlighting that the actions taken  
281 so far have not been sufficient to avoid breaching the limits.

282 We have illustrated the methodology in a simulation study. The simulation study is based on  
283 statistical models trained on historical temperature data to predict future temperatures. Our  
284 results are based on the assumption that no structural changes will occur in the future. In  
285 that sense, our results could be interpreted as a scenario in which no action is taken to reduce  
286 greenhouse gas emissions from the current levels.

287 The methodology can be easily extended to include different scenarios of future emissions and  
288 more complex models of the climate system. Climate models such as MAGICC already provide  
289 a range of possible outcomes for future temperatures; our methodology can be easily applied to  
290 these models. We encourage climate model developers to include the probabilities of breaching  
291 the limits in their reports.

## 292 **Reproducibility**

293 The code used to perform the simulation study is available in a [Jupyter notebook in the supple-](#)  
294 [mentary material](#). The code is written in Julia (Bezanson et al. 2017). The Julia programming  
295 language is a high-level and high-performance language for technical computing. Additional  
296 packages used in the simulation study are the `DataFrames.jl` package for data manipula-  
297 tion (Bouchet-Valat and Kamiński 2023), the `MarSwitching.jl` package for Markov-switching  
298 models (Dadej 2024), the `LongMemory.jl` package for long-range dependent models (J. E. Vera-

299 Valdés 2024), the `CSV.jl` package to read and write CSV files (Quinn et al. 2024), and the  
300 `Plots.jl` package for plotting (Brelhoff 2024).

301 The code is well documented and includes comments to explain the different steps of the  
302 simulation study. The code is open-source and can be freely used and modified. We encourage  
303 other researchers to use the code to reproduce our results and to extend the methodology to  
304 other datasets and models.

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## 457 **Supplementary material**

458 The supplementary material contains additional information on the models used in the simula-

459 tion study. The components of the models are described in detail.

460 **Trend models**

461 We consider three trend specifications for modeling the global mean temperature anomaly: a  
 462 linear trend model, a quadratic trend model, and a linear trend allowing for a break. The  
 463 models are given by:

- 464 • Linear Trend:  $y_t = \beta_0 + \beta_1 t + \gamma ONI_t + \epsilon_t$ ,
- 465 • Quadratic Trend:  $y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \gamma ONI_t + \epsilon_t$ ,
- 466 • Trend with Break:  $y_t = \beta_0 + \beta_1 t + \beta_2 I_{t>t_0} + \gamma ONI_t + \epsilon_t$ .

467 Above,  $y_t$  is the global mean temperature anomaly at time  $t$ ,  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$  are the trend  
 468 coefficients,  $\gamma$  is the coefficient of the El Niño effect,  $ONI_t$  is the variable that models the El  
 469 Niño events, and  $\epsilon_t$  is the error term. As described in the following, the error term is assumed  
 470 to have long-range dependence. The variable  $I_{t>t_0}$  is an indicator variable that takes the value  
 471 1 if  $t > t_0$  and 0 otherwise. The break point  $t_0$  is estimated from the data.

472 The models are estimated on the historical temperature data. The best model is selected based  
 473 on the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) (Akaike  
 474 1974; Schwarz 1978). For each realization, the model with the lowest AIC and BIC is considered  
 475 the best model and is used to predict future temperatures.

476 For example, the AIC and BIC for the trend models fitted to realization 10 are presented in  
 477 Table 2.

Table 2: Information criteria for model selection.

Model	AIC	BIC
Linear Trend	-5613.2	-5596.64
Quadratic Trend	-6551.17	-6529.09
Trend with Break	-6627.33	-6605.25

478 The estimated coefficient confidence intervals are used to simulate future values of the tem-  
 479 perature anomaly. The confidence intervals are obtained from the coefficients' (asymptotic)  
 480 distribution. Under normally distributed error term, the coefficient estimators are normally  
 481 distributed with mean and variance given by the following formula:

$$\hat{\beta} \sim N(\beta, \sigma^2(X'X)^{-1}),$$

482 where  $\hat{\beta}$  are the estimates,  $\beta$  are the true coefficients,  $\sigma^2$  is the variance of the error term,  
 483 and  $X$  is the design matrix. In case of non-normal error term, the coefficient estimators are  
 484 asymptotically normal using the central limit theorem under mild conditions (Wooldridge 2010).

## 485 El Niño Southern Oscillation (ENSO) model

486 El Niño Southern Oscillation (ENSO) is a natural climate phenomenon that influences global  
 487 temperature. It is characterized by periodic warming of sea surface temperatures in the central  
 488 and eastern equatorial Pacific Ocean. It is observed every 2-7 years and can last from 9 months  
 489 to 2 years.

490 Modeling the El Niño effect is crucial for predicting future temperatures. To control for the El  
 491 Niño effect, we include the Oceanic Niño Index (ONI) as a covariate in the models as described  
 492 above. The ONI is an indicator for monitoring the ENSO. El Niño conditions are present when  
 493 the ONI is +0.5 or higher. Oceanic La Niña conditions exist when the ONI is -0.5 or lower.

494 One complication with the El Niño effect is that it is difficult to predict. The El Niño events  
 495 are highly variable and can have different intensities. The El Niño effect can also interact with  
 496 other climate phenomena, such as the Indian Ocean Dipole and the Madden-Julian Oscillation.  
 497 This makes it challenging to model the El Niño effect accurately [see e.g.; Thirumalai et al.  
 498 (2024); Ham, Kim, and Luo (2019); L'Heureux et al. (2020); Hassanibesheli, Kurths, and  
 499 Boers (2022)]. In this study, we use a simple model to capture the El Niño effect. The model  
 500 is based on the historical ONI data and is used to simulate future ONI values.

501 The dynamics of the ONI are modeled using a Markov-switching model (Hamilton 1989). The  
 502 Markov-switching model is a regime-switching model that allows for the presence of different  
 503 regimes in the data. The model is given by:

$$ONI_t = \beta_j + \epsilon_{j,t},$$

504 where  $\beta_j$  is the coefficient for the  $j$ -th regime, and  $\epsilon_{j,t}$  is the error term with variance  $\sigma_j^2$ . A  
 505 latent state at time  $t$ ,  $s_t$ , indicates the regime. The dynamics of  $s_t$  are governed by a Markov

506 process:

$$Pr(s_t = j | s_{t-1} = i, s_{t-2}, \dots, s_1) = Pr(s_t = j | s_{t-1} = i) = p_{ij},$$

507 where  $p_{ij}$  is the transition probability from state  $i$  to  $j$ .

508 Note that the probability distribution of  $s_t$  given the entire path  $\{s_{t-1}, s_{t-2}, \dots, s_1\}$  depends  
509 only on the most recent state  $s_{t-1}$ .

510 In historical data, the effect can be estimated using maximum likelihood estimation and the  
511 expectation-maximization algorithm (Hamilton 1990). For forecasting, the effect is simulated  
512 using a stochastic process taking into account the probability of each regime.

513 To determine the number of regimes, we use the AIC and BIC. We consider a range of possible  
514 regimes and select the number of regimes that minimize the AIC and BIC. Table 3 shows the  
515 AIC and BIC for the ONI data. Only odd numbers of regimes are considered to ensure that  
516 the model includes both El Niño and La Niña events and neutral conditions.

Table 3: Information criteria for model selection.

Regimes	AIC	BIC
3-regimes	2438	2504
5-regimes	2342	2507
7-regimes	1394	1703

517 Hence, the number of states in the Markov-switching model is seven. The seven states are  
518 chosen to correspond to the different phases of the ENSO cycle ranging from very strong El  
519 Niño, strong El Niño, moderate El Niño, neutral, moderate La Niña, strong La Niña, to very  
520 strong La Niña.

### 521 Long-range dependent error term

522 Long-range dependent models imply that past values of the series have a long-lasting effect  
523 on the current value. It describes the tendency for successive values to remain close to each  
524 other or to be dependent. Interestingly, the notion of long-range dependence originated in the  
525 analysis related to climate data in the pioneering work of Hurst (1956) on the Nile River minima.  
526 Hurst determined that a dam built to control river flow should be designed to withstand the  
527 worst-case scenario. The worst-case scenario was determined by the long-range dependence in

528 the data. Years with high minima were likely to be followed by years with high minima. This  
 529 phenomenon is known as the Joseph effect. This is due to Joseph's interpretation in the Old  
 530 Testament of Pharaoh's dream, which predicted that seven years of plenty would be followed  
 531 by seven years of famine.

532 A long-range dependent model can be written as:

$$y_t = \sum_{j=1}^{\infty} \phi_j y_{t-j} + \epsilon_t,$$

533 where  $\epsilon_t$  is an i.i.d. process. The coefficients  $\phi_j$  decay hyperbolically (slowly) to zero as  $j$   
 534 increases. In contrast, the coefficients of standard models decay exponentially to zero.

535 The temperature series exhibit long-range dependence. In the context of breaching the limits  
 536 set out by the PA, the long-range dependence in the data is crucial since it affects the forecasted  
 537 temperature rise.

538 One likely explanation behind the presence of long-range dependence in the data is aggregation  
 539 (Clive W. J. Granger 1980; Zaffaroni 2004; Haldrup and Vera-Valdés 2017). The global mean  
 540 temperature anomaly is an aggregate of temperature data from different regions. The aggrega-  
 541 tion process can lead to long-range dependence in the data. To account for this property, we  
 542 model the error term in the trend models as a long-range dependent process.

543 We used the exact local Whittle estimator to estimate the long-range dependence in the data  
 544 (Shimotsu and Phillips 2005). The exact local Whittle estimator is a semi-parametric estima-  
 545 tor that estimates the long-range dependence parameter by maximizing the modified Whittle  
 546 likelihood function originally proposed by Künsch (1987).

547 The exact local Whittle estimator minimizes the function given by:

$$R(d) = \log \left( \frac{1}{m} \sum_{k=1}^m I_{\Delta^d}(\lambda_k) \right) - \frac{2d}{m} \sum_{k=1}^m \log(\lambda_k),$$

548 where  $I_{\Delta^d}(\lambda_k)$  is the periodogram of  $(1 - L)^d x_t$ , where  $(1 - L)^d$  is the fractional difference  
 549 operator (C. W. J. Granger and Joyeux 1980; Hosking 1981),  $\lambda_k = e^{i2\pi k/T}$  are the Fourier  
 550 frequencies, and  $m$  is the bandwidth parameter.

551 The exact local Whittle estimator is consistent and asymptotically normal. The long-range



552 dependence parameter is estimated for each realization separately. The estimated parameter is  
553 then used to simulate the error term in the models.

### 554 **Alternative data sources**

555 The simulation study is based on the HadCRUT5 dataset. However, the methodology can be  
556 easily extended to include other datasets. For example, the GISTEMP and Berkeley Earth  
557 datasets (GISTEMP 2020; R. A. Rohde and Hausfather 2020) provide alternative temperature  
558 anomalies data.

559 The GISTEMP dataset is produced by the NASA Goddard Institute for Space Studies and  
560 provides global temperature anomalies data from 1880. The results using the GISTEMP dataset  
561 are presented in Figure 8 and are summarized in Table 4a. The results are based on the  
562 simulation study presented in the [supplementary Jupyter notebook](#).

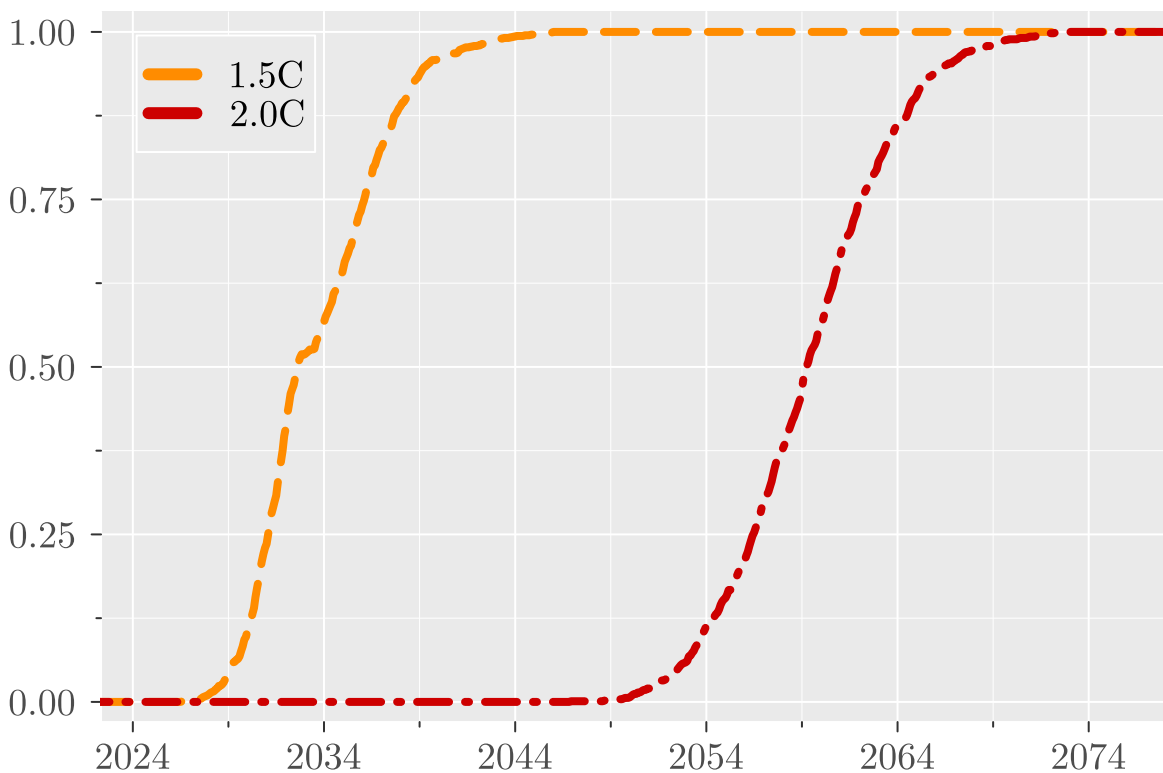


Figure 8: Proportion of scenarios that breach the 1.5°C and 2°C thresholds for the GISTEMP temperature anomalies for each month. The figure considers 1000 scenarios, each based on the best-fitting model for each realization, with five simulations for El Niño as an exogenous variable each.

563 Source: [Breaching 1.5°C: Give me the odds](#)

564 The results for the GISTEMP dataset show that the probability of breaching the 1.5°C limit is  
565 already greater than zero for May of 2027. Moreover, the probability of breaching it is greater  
566 than 99% by 2043. The results are in line with the results obtained using the HadCRUT5  
567 dataset.

568 The Berkeley Earth dataset is produced by the Berkeley Earth project and provides global  
569 temperature anomalies data from 1850. The results using the Berkeley Earth dataset are  
570 presented Figure 9 and are summarized in Table 4b. The results are based on the simulation  
571 study presented in the [supplementary Jupyter notebook](#).

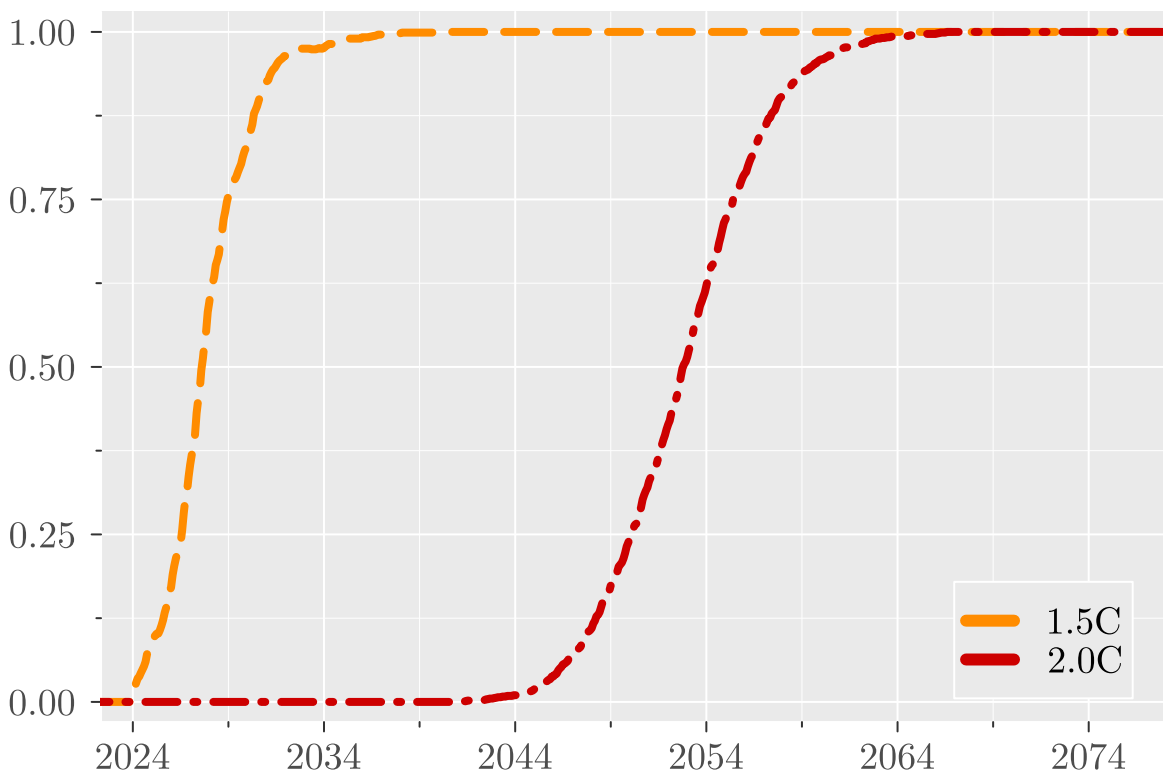


Figure 9: Proportion of scenarios that breach the 1.5°C and 2°C thresholds for the Berkeley Earth temperature anomalies for each month. The figure considers 1000 scenarios, each based on the best-fitting model for each realization, with five simulations for El Niño as an exogenous variable each.

572 Source: [Breaching 1.5°C: Give me the odds](#)

573 The results for the Berkeley Earth dataset show that the probability of breaching the 1.5°C  
574 limit is already greater than zero for September of 2024. Moreover, the probability of breaching  
575 it is greater than 99% by 2036. The results show a more rapid increase in the probability of

Table 4: Months to breach the 1.5°C and 2°C thresholds for alternative temperature anomalies datasets at a given probability level.

(a) GISTEMP dataset		(b) Berkeley Earth dataset			
Probability level and period	1.5°C threshold	2°C threshold	Probability level and period	1.5°C threshold	2°C threshold
Above 0%, 20-years avg.	2027-05-01	2047-09-01	Above 0%, 20-years avg.	2024-09-01	2041-10-01
Above 50%, 20-years avg.	2033-06-01	2060-05-01	Above 50%, 20-years avg.	2028-06-01	2053-08-01
Above 99%, 20-years avg.	2043-12-01	2071-02-01	Above 99%, 20-years avg.	2036-01-01	2063-08-01
Above 0%, 30-years avg.	2033-02-01	2052-09-01	Above 0%, 30-years avg.	2029-09-01	2046-05-01
Above 50%, 30-years avg.	2039-06-01	2065-05-01	Above 50%, 30-years avg.	2033-12-01	2058-10-01
Above 99%, 30-years avg.	2048-10-01	2075-02-01	Above 99%, 30-years avg.	2040-01-01	2068-11-01

576 breaching the 1.5°C limit compared to the HadCRUT5 dataset.